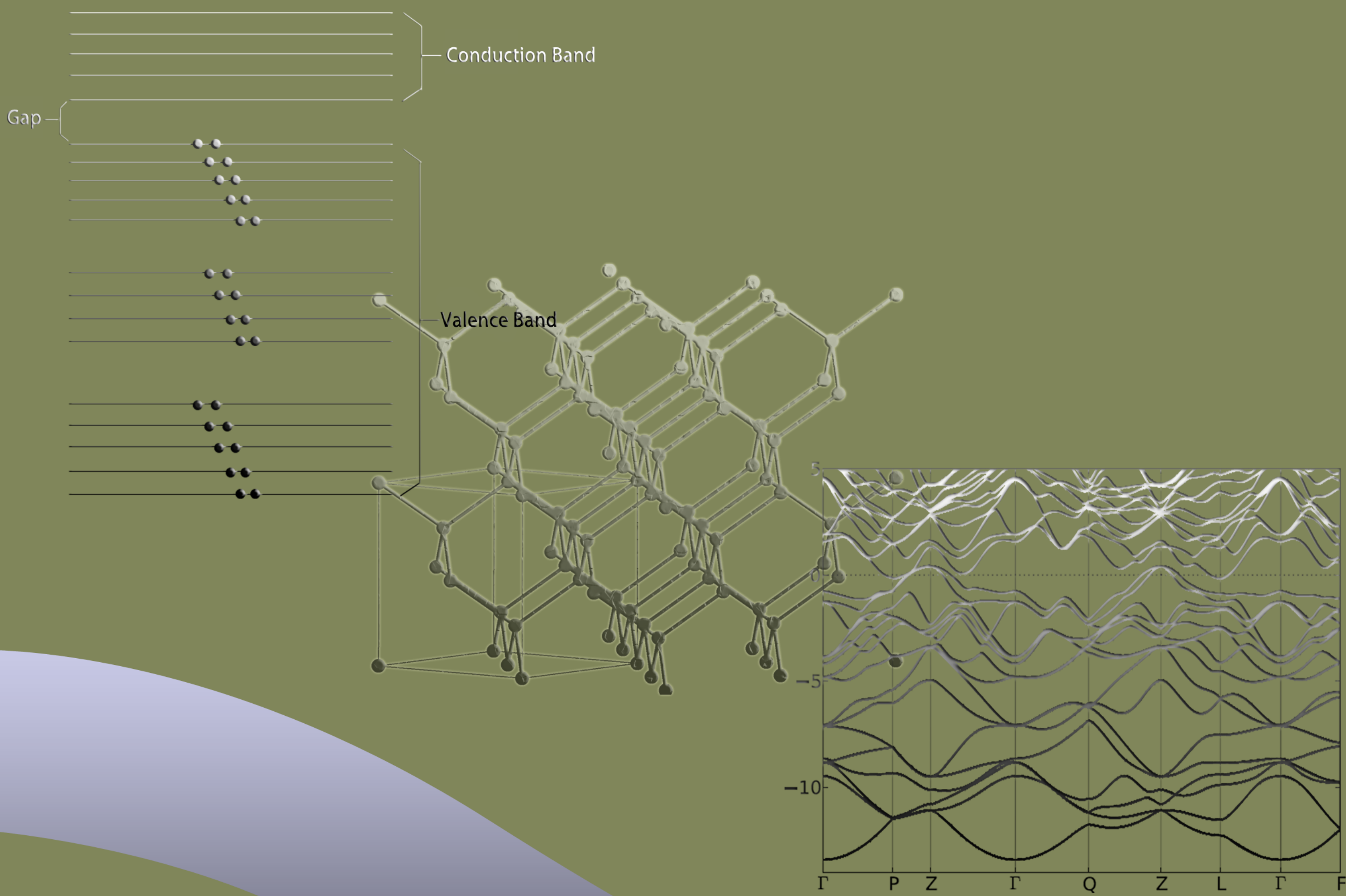
Queen Mary, University of London

School of Physics and Astronomy

About Coriolis Force



In this article, the motion of an object in the rotation reference framework and basics about Coriolis force will be discussed. When we talk about any motion in a rotation framework, we should firstly notice that the rotation reference is not inertial. Just like for any non-inertial reference, the Newton's second law cannot be directly used. For example, when we are trying bungee jumping, if we select the ground as reference, then it is natural to have the equation: $\mathbf{a} = m\mathbf{g}/m = \mathbf{g}$, which means our acceleration is $\mathbf{g}$ relative to the ground. However if we choose the bungee rope as the reference, we are actually stationary relative to the rope. But this time we still have the gravity, how can we then keep stationary in the new reference framework? Why? The reason is that the second framework we choose is non-inertial, where the normal Newton's second law cannot be directly used. If we still want to use the Newton's second law in a non-inertial framework, we should introduce an 'artificial' force, which is always called inertia force. If we have our non-inertial framework with acceleration of $\mathbf{a}$, the way we introduce the inertia force is through defining the inertia force correspondingly as $\mathbf{F}' = -m\mathbf{a}$. Given the definition of inertia force, we can easily apply Newton's second law on our previous bungee jumping problem (set bungee rope as our reference) to have: $\mathbf{F}' + m\mathbf{g} = -m\mathbf{g} + m\mathbf{g} = 0$, which then means we are stationary in the non-inertial reference framework.

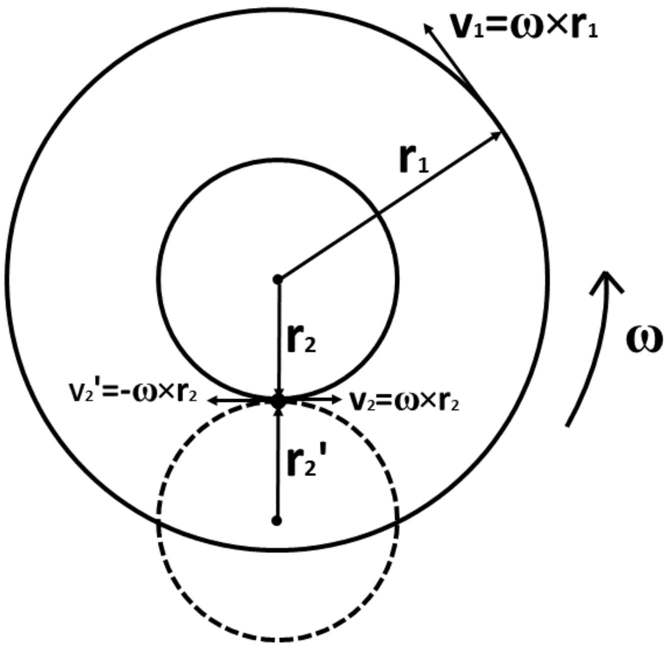


Figure 1. The illustration for rotation framework.

Keeping the definition of inertia force in mind, let's then have a look at the rotation framework. As we all know, any rotation framework is not inertial since we always have the centripetal acceleration (of course, if we choose an inertial framework to observe the rotating system). Fig. 1. shows a rotation framework, which basically depicts the condition when we have a plate rotating (angular velocity is ω) on the surface of a desk (without friction). Now let's put a block (with mass m) on the rotating plate, and again we ignore the friction between the plate and the block. Moreover, for the following discussion based on the system shown by Fig. 1., we only take the horizontal direction into consideration since gravity and reaction force makes equiblium in vertical direction. Then do you think the block on the plate will move if we observe it from the ground? No, it will never move relative to the ground since we don't have any force exerted on the block in horizontal direction. What about choosing the rotating plate as our reference? When we choose the rotating plate as the reference, the block on the plate is not stationary any more. Actually, it is 'rotating' relative to the plate as well. It is easy to imagine the block is moving relative to the rotating plate, but why do we say it is 'rotating' as well? If we have a look at Fig. 1., and we say the distance from the block to the center of the plate is r_2, then we should have every point passing by the block (just under it) with the velocity of $\mathbf{v}_2 = \boldsymbol{\omega} \times \mathbf{r}_2$. Thus the velocity of the block relative to the point just under the block on the plate should be $\mathbf{v}_2^{'} = -\boldsymbol{\omega} \times \mathbf{r}_2$. Then we could imagine a virtual 'rotation' shown by dashed circle in Fig. 1., where we have the block always at the top of the virtual rotation. Without any further knowledge, we could also imagine the virtual rotation is with the same radius as r_2. Thus we should have $r_2^{'} = r_2$ in Fig. 1., since we could notice that there is some kind of symmetry in there. And later we will see that our intuition is correct. Now, let's go back to the rotating plate and calculate the acceleration of the point just under the block. The result should be $\mathbf{a} = -\omega^2\mathbf{r}_2$, and then we should have corresponding inertia force $\mathbf{F}^{'} = m\omega^2\mathbf{r}_2$. For the virtual rotation that we have imagined, the inertia force $\mathbf{F}^{'}$ is just the force we need to provide the centropetal acceleration, and that's the only horizontal force exerted on the block in the rotation reference framework. Since we already have the velocity $\mathbf{v}_2^{'} = -\boldsymbol{\omega} \times \mathbf{r}_2$ of the block relative to the plate, then we should have: $\mathbf{F}^{'} = m\omega^2\mathbf{r}_2 = m\frac{v_2^{'2}}{r_2^{'}}\hat{\mathbf{r}}_2 = m\frac{\omega^2 r_2^2}{r_2^{'}}\hat{\mathbf{r}}_2$ ($\hat{\mathbf{r}}_2$ is the unit vector corresponding to $\mathbf{r}_2$), from which we can easily tell that $r_2^{'} = r_2$. That means our previous intuition for the radius of the virtual rotation is correct. So basically, in the rotating framework given by Fig. 1., we have the block (which is stationary relative to the ground) 'rotating' relative to the plate, where the inertia force $\mathbf{F}^{'}$ provides the centripetal acceleration for the virtual rotation.

Then let's have a look at a little bit more complex system shown in Fig. 2., where we have the same rotating plate and the block on it. But this time, let's imagine the block is moving away from (Fig. 2. (a)) or towards (Fig. 2. (b)) the center of the plate with the velocity of $\mathbf{v}_r^{'}$. If observed from the ground, the block will be with constant velocity since we don't have any horizontal force exerted on it. However if we select the plate as our reference, the block is moving tangentially relative to the plate with the velocity of $\mathbf{v}_2^{'}$ as shown in Fig. 2. As we already know from the previous problem where the block is stationary relative to ground, the relative velocity $\mathbf{v}_2^{'}$ to the plate does depend on the distance from the block to the center of the plate. Thus in this case where we have the block moving towards or backwards the center of the plate, the changing of radius $\mathbf{r}_2$ in the previous problem leads to the changing of relative velocity $\mathbf{v}_2^{'}$ of the bolck to the plate. For Fig. 2. (a), $\mathbf{v}_2^{'}$ keeps increasing since $\mathbf{r}_2$ is increasing, and the other way round for the case shown in Fig. 2.

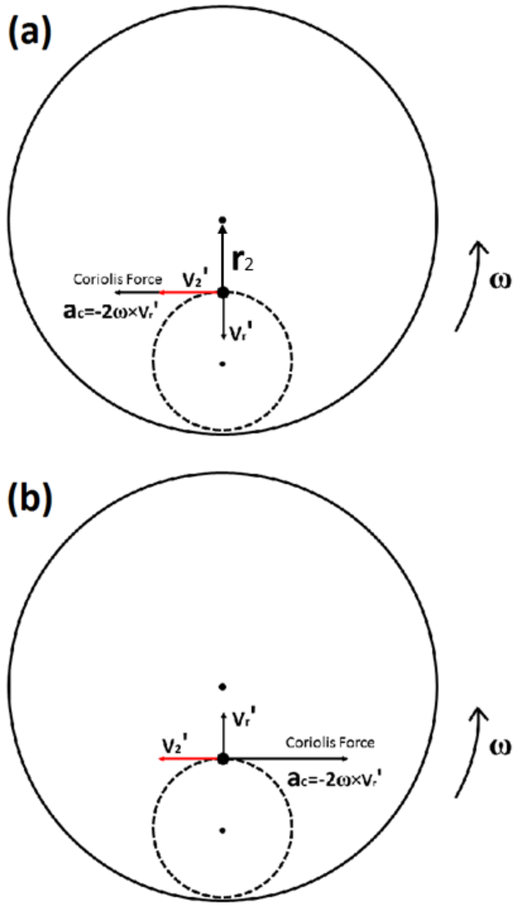


Figure 2. The illustration for rotation framework, where the object is moving.

(b). This means in the rotation framework (the plate), we should have corresponding force, which could provide the acceleration needed for the changing of the relative tangential velocity $\mathbf{v}_2'$. And that force is just part of the Coriolis force, which is, obviously, inertia force. Here let's remember the expression 'part of' for now, and we will come back later to discuss why we say 'part of the Coriolis force' here. Let's recall another inertia force in the system, which is the centrifugal force $\mathbf{F}' = m\omega^2\mathbf{r}_2$. And for the imagined virtual rotation of the block (see the previous problem), we always have: $\mathbf{F}' = m\omega^2\mathbf{r}_2 = m\frac{v_2'^2}{r_2}\hat{\mathbf{r}}_2 = m\frac{\omega^2 r_2^2}{r_2}\hat{\mathbf{r}}_2 = m\omega^2\mathbf{r}_2$, which means no matter where the block is, the centrifugal force $\mathbf{F}'$ is always totally spent on providing centripetal acceleration for the virtual rotation. Thus it is natural that the centripetal or centrifugal velocity of the block does not change. Now let's calculate the relative tangential acceleration of the block to the plate. Here we only discuss Fig. 2. (a), and Fig. 2. (b) is quite similar. During time interval Δt after the block leaves the center of the plate, we should have: $r_2 = v_1'\Delta t$, and then we have $v_2' = \omega r_2 = \omega v_1'\Delta t$. Then we could get the magnitude for the tangential accerlaeration $a_c = \omega v_1'$. However if we notice the Coriolis force labeled in Fig. 2., we should have the magnitude for Coriolis acceleration as: $a_c = 2\omega v'$. Puzzled? Why we have two different a_c here? Why we have a factor '2' for real Coriolis acceleration? It seems they should be the same, both of which are for the relative tangential acceleration of the block! Now let's go back to the 'part of' expression that we didn't talk about just now. Let's take a look at the block again, and we first observe it from the ground. The velocity vector $\mathbf{v}_1'$ always point straightforward down in this case. Then what about observing it from the perspective of the rotating plate? Is $\mathbf{v}_1'$ still always going down? No! Because the plate is rotating, thus the velocity vector of the block is always changing its direction relative to the plate! Then what? We have $\mathbf{v}_1'$ changing its direction from time to time, thus we should have a tangential acceleration (to change the direction of $\mathbf{v}_1'$, again relative to the plate), which is just another part of the Coriolis acceleration! This part of tangential acceleration together with the other part tangential acceleration that we directly derived above contributes to the final factor '2' in Coriolis acceleration.

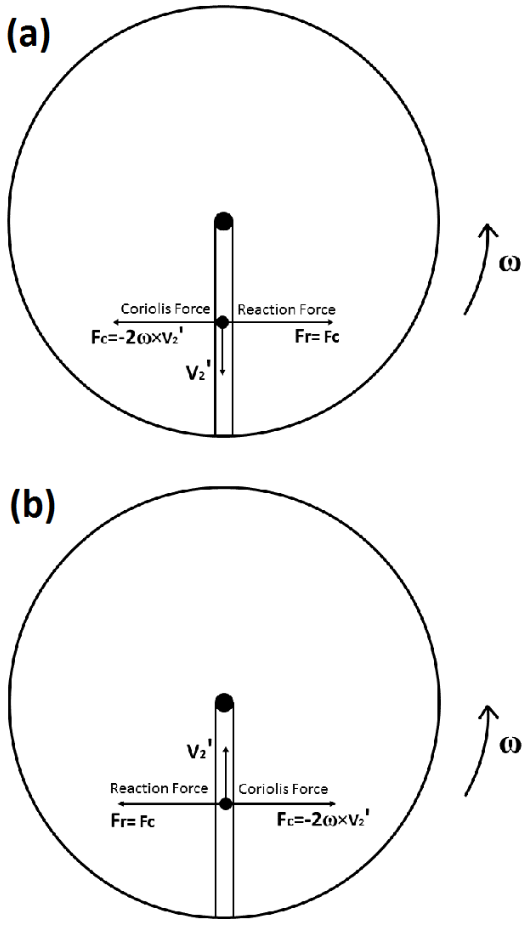


Figure 3. Illustration for object motion on rotating plate, where we have a tube through the center of the plate to restrict the object.

Now let's go to next system as shown in Fig. 3., which is again a bit more complex than the previous one. When the block is restricted within the tube, it has to follow the rotation of the plate, which means there is no relative motion of the block to the plate in tangential direction. And also the direction of centrifugal velocity $\mathbf{v}_2'$ (relative to the plate) does not change, either. So basically we have equilibrium in the tangential direction, which should attribute to the resistance against the Coriolis force from the reaction force as shown in Fig. 3. (a) & (b). Since we don't have the relative motion of the block to the plate in tangential direction, we then don't have the virtual rotation as in the previous two problems. That means the centrifugal force $\mathbf{F}'$ is totally spent on accelerating the block in centrifugal direction, which then makes $\mathbf{v}'_2$ larger and larger in Fig. 3. (a), or smaller and smaller in Fig. 3. (b).

Before going on the discussion, let's review all the previous problems, and this time we look at them from the perspective of energy. In the rotation framework, we have corresponding inertia centrifugal force, which tends to push any objects in the rotation framework far away from the center. That means we have potential in the rotation system, which is the potential for any objects in the rotation system to go away from the center. It is just like what we have on earth where we have the gravity which then tends to pull everything down to the earth. And if an object is located at high altitude, it has the potential to fall down and transform the potential energy to kinetic energy. For the first two problems in this article, we have the potential there, but why didn't the potential energy transform to kinetic energy? What prevents that transformation (it seems that we don't have any resistance against the centrifugal force there)? It is the virtual rotation that we imagined for the first two problems that 'resists' the centrifugal force to prevent it from pushing our object away from the center! Actually, it is just like the space station, when we are rotating around our earth, we cannot 'feel' the gravity since the whole gravity is spent on providing us with the centripetal acceleration. However for the third problem discussed above, there is resistance from the wall of the tube to prevent the block from moving relative to the plate in tangential direction (actually, it prevents the virtual rotation as we imagined for the first two problems). Thus the centrifugal force has nowhere to spend, and it 'has to' be used on accelerating the block along the centrifugal or centripetal direction. Then we have the potential energy in the rotation framework transforming to kinetic energy (Fig. 3.) or the other way round (Fig. 3. (b)).

Finally, let's look at another problem in the rotation frame, which is shown in Fig. 4. Here let's imagine a string going through the center of the plate and tied to the block. The plate is again rotating with the angular velocity of $\boldsymbol{\omega}$, and at the same time we pull the string from the other end to drag the block towards the center of the plate. At the beginning, the string and the block rotates together with the plate (the string and block are stationary to the plate), which then gives the velocity $\mathbf{v}_1 = \boldsymbol{\omega} \times \mathbf{r}_1$ of the block relative to the ground (so we should have the tension $F_T = m\frac{v_1^2}{r_1}$ in the string). Then we starts to pull the string harder to drag the block to the center of the plate, keeping constant velocity v'_2. Then let's think about the tangential velocity $\mathbf{v}_1$, will it change its magnitude? Of course not, since we don't have any tangential force exerted on the block! Then we can understand why we need to pull 'harder' since our radius of the rotation for the block is reduced as we pull it towards the center (remember $F_T = m\frac{v_1^2}{r}$). Let's now turn into our rotating plate as our reference. Firstly, we should have the centrifugal force (which is inertia force) in equiblium with

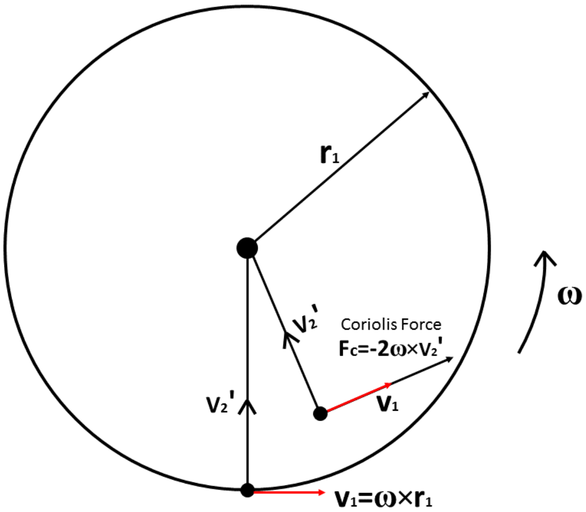


Figure 4. The illustration for the motion of an object on the rotating plate. Here the object is tied to a string through the center of the plate.

the tension in the string, keeing the block moving in contant velocity in the centripetal direction. Then for the tangential direction, at the beginning, the velocity of the block relative to the plate is 0. As the block goes towards the center of the plate, its tangential velocity relative to the ground does not change as we discussed above, but the velocity of the point (just under the block) on the rotating plate does change. After time interval Δt from the beginning, the distance between the block and the center of the plate becomes: $r_1 - v_2^{'}\Delta t$. Just under the block, the velocity of the point on the plate then is: $\frac{r_1 - v_2^{'}\Delta t}{r_1}v_1$. Thus the relative velocity of the block to the plate is: $v_1 - \frac{r_1 - v_2^{'}\Delta t}{r_1}v_1 = \frac{v_2^{'}\Delta t}{r}v_1 = \omega v_2^{'}\Delta t$. Again we use the formula of Criolis acceleration to give the relative tangential acceleration of the block to the plate: $a_c = 2\omega v_2^{'}$. Again we have the factor '2' here and if we calculate the relative tangential velocity of the block to the plate using the Coriolis acceleration, it will again contradict with the result that we derived above. Why? Let's recall our explanation for the second problem, where we say the Coriolis acceleration contains two parts. The first part is spent on increasing the magnitude of tangential relative velocity of the block to the plate, which accounts for the result $\omega v_2^{'}\Delta t$ derived above. The other part is spent on changing the direction of the centripetal velocity $\mathbf{v}_2^{'}$, as we can see from Fig. 4. the direction changing of $\mathbf{v}_2^{'}$.

So basically, Coriolis force is the inertia force in non-inertial reference framework to account for the changing of relative speed in tangential direction. If we look at our object in an inertia framework, there will never be Coriolis force at all! But why do we bother with Coriolis force? Why don't we always choose inertial force to make our life easier? The fact is, our earth is just a rotation framework! We stand on earth, thus it is natural to select our earth as the reference, and sometimes, we have to consider the effect of Coriolis force, e.g. the formation of tornado.